Question 1

A function g with domain R has the following properties.

- $g'(x) = x^2 - 2x$
- the graph of $g(x)$ passes through the point $(1, 0)$

$g(x)$ is equal to

A. $2x - 2$

B. $\frac{x^3}{3} - x^2$

C. $\frac{x^3}{3} - x^2 + \frac{2}{3}$

D. $x^2 - 2x + 2$

E. $3x^3 - x^2 - 1$

(i) Identify the correct response and show working.

(ii) Which response anticipates someone might incorrectly use differentiation instead of anti-differentiation?

(iii) What is immediately wrong with D?

(iv) Change the y-coordinate of the point (1, 0) in the question so that B would become correct.

[2 + 1 + 1 + 2 = 6 marks]

Question 2

A cubic function has the rule $y = f(x)$. The graph of the derivative function f' crosses the x-axis at $(2, 0)$ and $(-3, 0)$. The maximum value of the derivative function is 10.

The value of x for which the graph of $y = f(x)$ has a local maximum is

A. -2

B. 2

C. -3

D. 3

E. $-\frac{1}{2}$

(i) Identify the correct response and show working.

(ii) What type of function must f' be? Draw a possible sketch of f.

(iii) Suggest a reason as to why someone might incorrectly choose E.

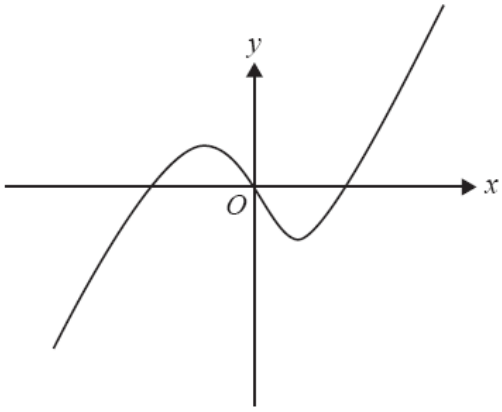
(iv) Change one word in the question that would make C the correct response.

(v) Explain the relevance of the word 'local' in the question?

[2 + 2 + 2 + 1 + 1 = 8 marks]

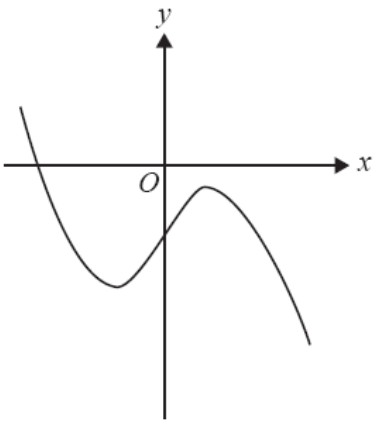
Question 3

The graph of the gradient function $y = f'(x)$ is shown below.

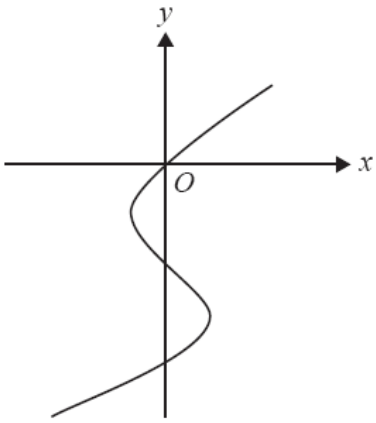


Which of the following could represent the graph of the function $f(x)$?

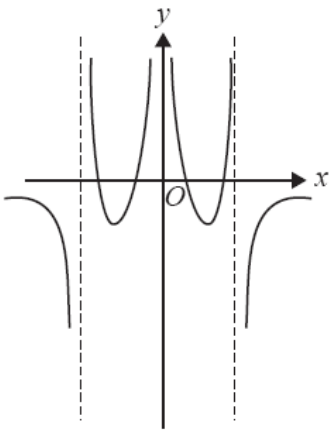
A.



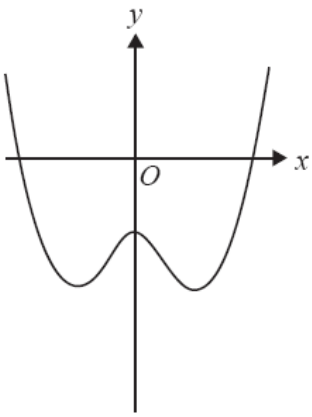
B.



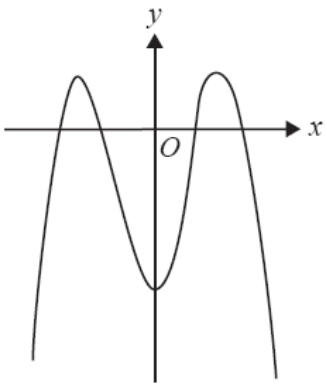
C.



D.



E.



(i) What is immediately wrong with B?

(ii) What is immediately wrong with C?

(iii) What could be a reason that someone would mistakenly choose E?

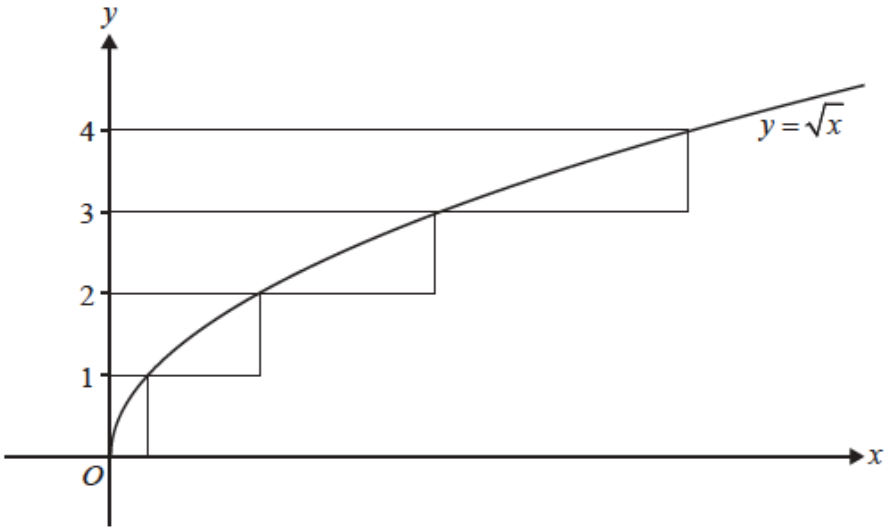
(iv) Sketch a possible graph of $f'(x)$ that would make A correct.

[1 + 1 + 1 + 2 = 5 marks]

Question 4

The graph of $f: R^{+} \cup \{0\} \to R,\ f(x) = \sqrt{x}$ is shown below.

In order to find an approximation to the area of the region bounded by the graph of f, the y-axis and the line $y = 4$, Zoe draws four rectangles, as shown, and calculates their total area.



Zoe's approximation to the area of the region is

A. 14

B. 21

C. 29

D. 30

E. $\frac{64}{3}$

(i) **Label** the coordinates where the rectangles touch the curve and **shade** the actual area that the rectangles are being used to approximate.

(ii) Which response is, in fact, the *exact* area bound by the y-axis, the curve and the line $y = 4$, and not the area *approximated* by the rectangles as asked for in the question. Show working.

(iii) Two of the responses would result from including only 3 of the 4 rectangles. Which are they?

(iv) Which is the correct response to the actual question?

(v) Find a trapezoidal approximation for the same area.

[2 + 2 + 1 + 1 + 2 = 8 marks]

Question 5

Consider the region bounded by the x-axis, the y-axis, the line with equation $y = 3$ and the curve with equation $y = \log_e(x - 1)$.

The exact value of the area of this region is

A. $e^{-3} - 1$

B. $16 + 3\log_e(2)$

C. $3e^3 - e^{-3} + 2$

D. $e^3 + 2$

E. $3e^2$

This question was written for the 2009 Exam.

(i) Sketch the curve and shade the region as described in the question.

The Examiner's Report cites the following equation as a solution, indicating that D is the correct response.

$$\int_0^3 \left(e^x + 1\right)dx = e^3 + 2$$

(ii) Sketch the examiner's integral, labelling the area referred to as $e^3 + 2$ and explain why it is the same as the area described in the original question.

(iii) Using this same approach, find the exact area bound by the *x*-axis, the *y*-axis, the curve with equation $y = \log_e\left(\frac{x}{2}\right)$ and the line with equation $y = -2$. Show your working.

[1 + 3 + 3 = 7 marks]

Question 6

The average value of the function f: $[0, 2\pi] \rightarrow R, f(x) = \sin^2(x)$ over the interval $[0, a]$ is 0.4. The value of a, to three decimal places, is

A. 0.850
B. 1.164
C. 1.298
D. 1.339
E. 4.046

(i) Sketch the function.

(ii) Which response is simply a value of a for which the area under the curve over the interval $[0, a]$ is 0.4?

(iii) Rewrite the first sentence of the question as an equation. Hence use a CAS calculator to determine the correct response.

[1 + 2 + 2 = 5 marks]